Chapter 9

Controller Implementation

9.1 Controller Canonical Form

Lets look at the generic feedback control system in Fig. 9.1.

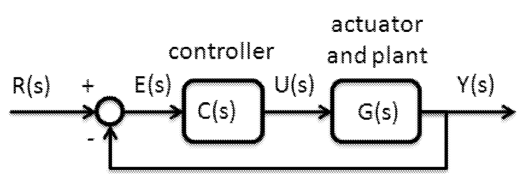

Figure 9.1: Generic feedback control system

where the controller $C(s)$ is always a rational polynomial of s as

$$C(s) = \frac{U(s)}{E(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \dot{+} a_m}{b_0 s^n + b_1 s^{n-1} + \dot{+} 1}; n \geq m \tag{9.1}$$

which can be rearranged as follows, where we have an expression for the control signal $u(s)$ in a configuration known as controller canonical form.

$$\begin{aligned} \{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1} s + 1\} U(s) &= \{a_0 s^m + a_1 s^{m-1} + \ldots + a_m\} E(s) \\ U(s) &= \{a_0 s^m + a_1 s^{m-1} + \ldots + a_m\} E(s) \\ &\quad - \{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1} s\} U(s) \end{aligned} \tag{9.2}$$

9.2 Analog Implementation

The controller canonical form obtained in (9.2) can be physically realized as shown in Fig.9.2, which shows that the controller is an arrangement of $(m+n)$ differentiators, $(m+n)$ adders and one subtracter. Lets look at how these mathematical operations can be electronically devised for controller synthesis.

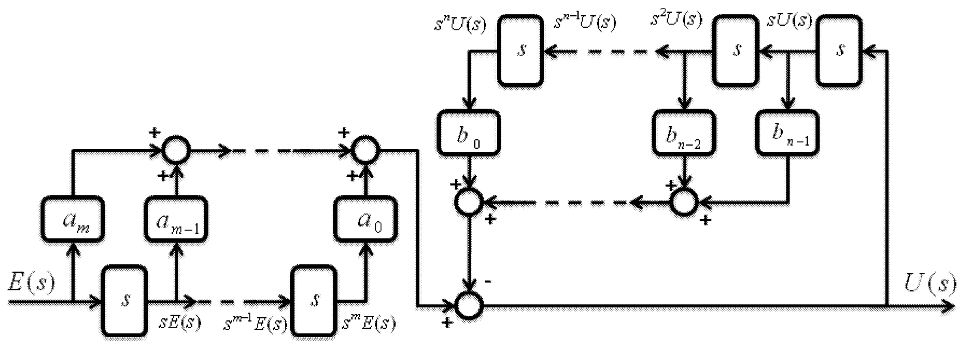

Figure 9.2: Synthesis of control input $U(s)$

9.2.1 Operational Amplifier in Controller Synthesis

The operational amplifier (OpAmp) symbolized by a triangle with two inputs and one output is a specially designed transistor circuit, which is able to perform mathematical operations such as addition, multiplications, differentiation and integration. In fact, using OpAmps, resistors, and capacitors it is possible to built controllers. Following two characteristics of the OpAmp allows it to perform these mathematical operations.

- OpAmp has a very high input impedance, therefore, it does not draw any current into it.
- OpAmp has no voltage difference between its two input terminals namely; noninverting input(+) and inverting input(-).

Positive Gain (non-inverting amplifier)

Figure 9.3 shows a gain (non inverting amplifier) circuit

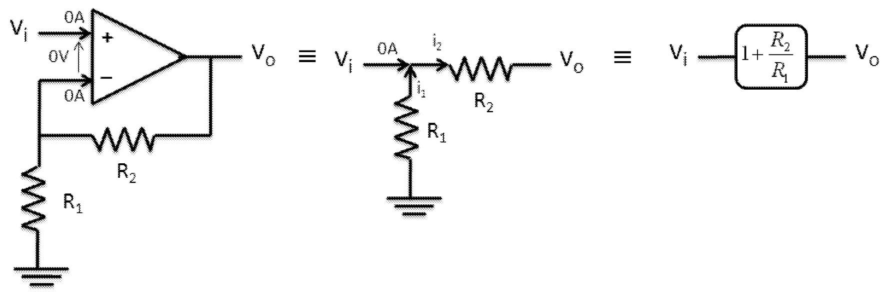

Figure 9.3: A positive gain constructed with an OpAmp

where input voltage (signal) is v_i and output signal is v_o. As there won't be any voltage difference between the two input terminals, -ve terminal will assume a voltage v_i, and as there won't be any current going into the OpAmp, current flows from the output terminal to the ground through the two resistors. Therefore,

$$\begin{aligned} \frac{v_o - v_i}{R_2} &= \frac{v_i}{R_1} \\ R_1(v_o - v_i) &= R_2 v_i \\ \frac{v_o}{v_i} &= \frac{R_1 + R_2}{R_1} \\ V_o(s) &= \frac{R_1 + R_2}{R_1} V_i(s) \end{aligned} \tag{9.3}$$

Negative Gain (inverting amplifier)

Figure 9.4 shows a -ve gain

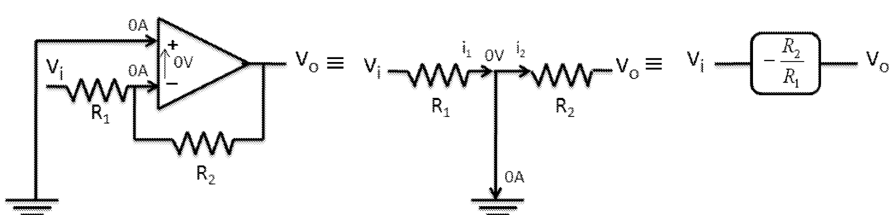

Figure 9.4: A -ve gain constructed with an OpAmp

Using the two OpAmp characteristics mentioned, we can identify currents and voltages as shown in the equivalent circuit. Current flows from v_i to v_o through the two resistors. Therefore,

$$\begin{aligned} \frac{v_i}{R_1} &= -\frac{v_o}{R_2} \\ \frac{v_o}{v_i} &= -\frac{R_2}{R_1} \\ V_o(s) &= -\frac{R_2}{R_1}V_i(s) \end{aligned} \tag{9.4}$$

Integrator with a Negative Gain

Figure 9.5 shows an integrator with a positive gain

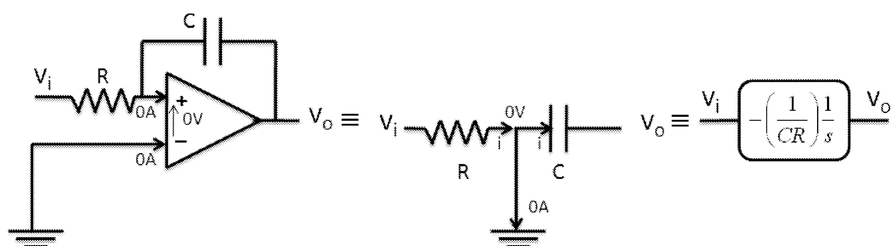

Figure 9.5: An integrator with a negative gain constructed with an OpAmp

As OpAmp does not suck any, the current flows from from v_i to v_o through thw two resistors. As +ve input terminal is grounded (0V), -ve input terminal will also be assumed 0V. Then we can write KCL for -ve input terminal as $\frac{v_i}{R} + C\frac{dv_o(t)}{dt} = 0$, and transform into Laplace domain as follows.

$$\begin{aligned} \frac{1}{R}V_i(s) + CsV_0(s) &= 0 \\ V_o(s) &= -\left(\frac{1}{CR}\right)\frac{1}{s}V_i(s) \end{aligned} \tag{9.5}$$

Differentiator with a Negative Gain

Figure 9.6 shows an integrator with a negative gain.

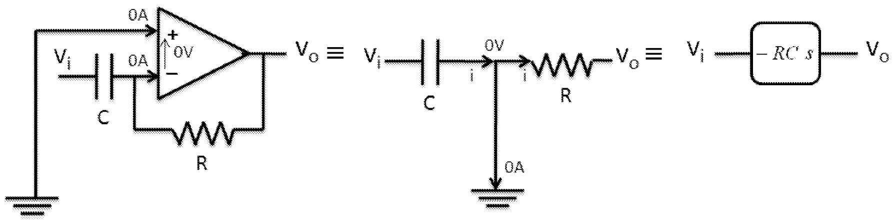

Figure 9.6: A differentiator with a negative gain constructed with an OpAmp

Grounding positive input terminal pulls the negative terminal to 0V. As no current flows into the OpAmp, current path is from v_i to v_o through the resistors. We can write KCL for the negative input terminal as $C\frac{dv_i(t)}{dt} + \frac{v_o(t)}{R} = 0$, which when transformed into Laplace domain

$$\begin{aligned} CsV_i(s) + \frac{1}{R}V_0(s) &= 0 \\ \frac{V_o(s)}{V_i(s)} &= -RC.s \end{aligned} \tag{9.6}$$

Error Signal with a Positive Gain

Figure 9.7 shows how error signal can be constructed using OpAmps.

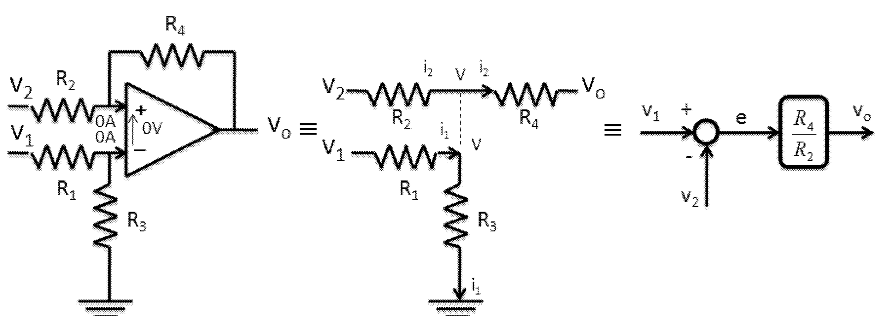

Figure 9.7: Determination of error signal with a positive gain constructed with an OpAmp

Both input terminals assume the same voltage v however, there won't be any current flowing between these terminals through the OpAmp. Therefore, we have two independent current paths as shown in the equivalent circuit. According to the first current path, we can write KCL for the common voltage node as follows

$$\begin{aligned} \frac{v_2 - v}{R_2} + \frac{v_o - v}{R_4} = 0 \\ \frac{v_2}{R_2} + \frac{v_o}{R_4} - \left(\frac{1}{R_2} + \frac{1}{R_4}\right) v = 0 \end{aligned} \tag{9.7}$$

From the other current path (a voltage divider) we can find an expression for the common voltage v as

$$v = v_1 \left(\frac{R_3}{R_1 + R_3}\right) \tag{9.8}$$

By substitution from (9.8) to (9.8) for v

$$\frac{v_2}{R_2} + \frac{v_o}{R_4} - \left(\frac{1}{R_2} + \frac{1}{R_4}\right)\left(\frac{R_3}{R_1 + R_3}\right) v_1 = 0 \tag{9.9}$$

If we select resistors such that $\left(\frac{1}{R_2} + \frac{1}{R_4}\right)\left(\frac{R_3}{R_1+R_3}\right) = \frac{1}{R_2}$ then (9.9) will become as follows

$$\begin{aligned} \frac{v_2}{R_2} + \frac{v_o}{R_4} - \frac{v_1}{R_2} &= 0 \\ v_o &= \frac{R_4}{R_2}(v_1 - v_2) \\ V_o(s) &= \frac{R_4}{R_2}\left[V_1(s) - V_2(s)\right] \end{aligned} \tag{9.10}$$

Summing Point (OpAmp Adder)

Figure 9.8 shows how two signals can be added using an OpAmp.

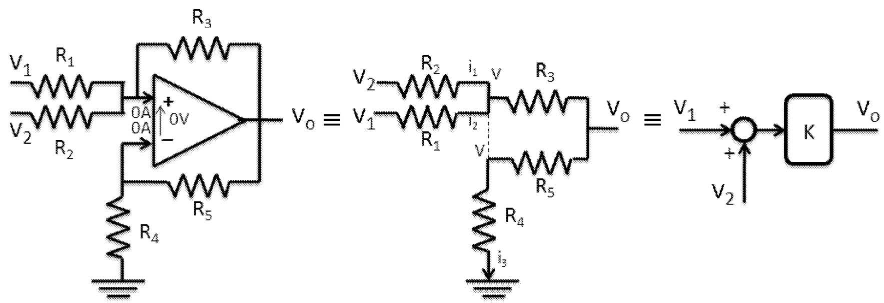

Figure 9.8: Summing point built using OpAmp adder circuit

As shown in the equivalent circuit, there are two current paths to the output from both input terminals. Therefore, for the +ve terminal we can write KCl as follows

$$
\begin{aligned}
\frac{v_1 - v}{R_1} + \frac{v_2 - v}{R_2} + \frac{v_0 - v}{R_3} &= 0 \\
\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_o}{R_3} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v &= 0 \qquad (9.11)
\end{aligned}
$$

The other current path, which is a potential divider from -ve terminal to output, we can find the following expression for v

$$v = \frac{R_4}{R_4 + R_5} v_o \qquad (9.12)$$

By substitution from (9.12) to (9.11) for v

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_o}{R_3} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{R_4}{R_4 + R_5}\right) v_o = 0 \qquad (9.13)$$

This expression can be used to build various adders. We can use $R_1 \neq R_2$ if we want to give different weights for v_1 and v_2. For simple addition, set $R_1 = R_2 = R_3 = R$, then

$$\frac{v_1 + v_2}{R} + \left(\frac{1}{R} - \frac{3}{R}\frac{R_4}{R_4 + R_5}\right) v_o = 0 \qquad (9.14)$$

If we further set $R_4 = R$ and $R_5 = 2R$ then,

$$\begin{aligned}
\frac{v_1 + v_2}{R} + \left(\frac{1}{R} - \frac{3}{R}\frac{2R}{3R}\right) v_o &= 0 \\
\frac{v_1 + v_2}{R} + \left(\frac{1}{R} - \frac{2}{R}\right) v_o &= 0 \\
v_1 + v_2 = v_0 & \\
V_1(s) + V_2(s) &= V_o(s) \qquad (9.15)
\end{aligned}$$

9.3 Digital Control

9.3.1 Discretization of the Controller Canonical Form

Controller canonical form in (9.1) can be written as the following differential equation.

$$b_0 \frac{d^n u(t)}{dt^n} + b_1 \frac{d^{n-1} u(t)}{dt^{n-1}} + \ldots + u(t) = a_0 \frac{d^m e(t)}{dt^m} + a_1 \frac{d^{m-1} e(t)}{dt^{m-1}} + \ldots + a_m e(t) \qquad (9.16)$$

This continuous-time differential equation can be approximated with a difference equation by taking samples of $u(t)$ as shown in Fig.9.9.

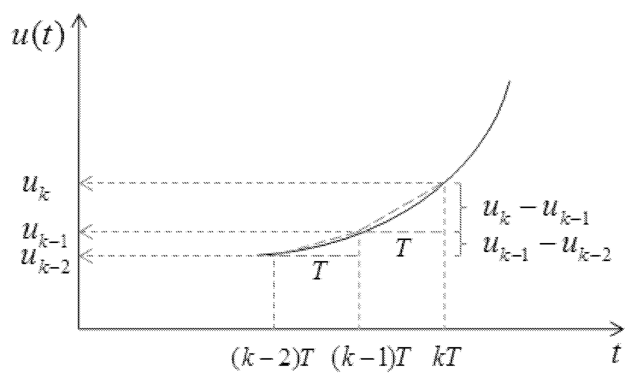

Figure 9.9: Sampling of $u(t)$ at T[s] interval

These samples can be used to determine approximate descrete-time equivalents to $u(t)$ and its derivatives as follows.

$$u(t) \approx u_k$$

$$
\begin{aligned}
\frac{du(t)}{dt} &\approx \frac{u_k - u_{k-1}}{T} \\
\frac{du^2(t)}{dt^2} &\approx \frac{u_k - 2u_{k-1} + u_{k-2}}{T^2} \\
&\vdots
\end{aligned} \tag{9.17}
$$

Similarly, descrete-time equivalents for $e(t)$ and its derivatives are as follows

$$
\begin{aligned}
e(t) &\approx e_k \\
\frac{de(t)}{dt} &\approx \frac{e_k - e_{k-1}}{T} \\
\frac{de^2(t)}{dt^2} &\approx \frac{e_k - 2e_{k-1} + e_{k-2}}{T^2} \\
&\vdots
\end{aligned} \tag{9.18}
$$

Using (9.17) and (9.18), the descrete-time canonical controller can be written as follows.

$$
\begin{aligned}
f(u_{k-1}, u_{k-2}, ..., u_{k-n}) + u_k = g(e_k, e_{k-1}, ..., e_{k-m}) \\
u_k = g(e_k, e_{k-1}, ..., e_{k-m}) - f(u_{k-1}, u_{k-2}, ..., u_{k-n})
\end{aligned} \tag{9.19}
$$

Actual implementation of this digital implementation is demonstrated in the next section through an example.

9.3.2 Digital Controller Implementation

In recent times, programmable ICs (integrated circuits), which are known as microcontrollers have gained popularity as they can be programmed with discrete-time controllers quite effectively. Once programmed, these microcontroller can implement the task of the controlling the plant. The most common microcontrollers are PIC [10] and Atmega [11], and they come integrated with their development software so that the user can choose a friendly programming language (eg. PIC BASIC, or PIC C), or learn how to program with the given language keywords. Once the program is written, debugged, and compiled correctly, it can be downloaded from the computer to the microcontroller either directly, or using a programmer through serial, parallel, or USB interfaces. The microcontroller programming environment is shown in Fig.9.10. Once the digital controller is downloaded onto the microcontroller it is unplugged from the programmer and plugged into the plant with proper connections of input pins to the reference $r(t)$ and response $y(t)$, and output pin to the actuator $u(t)$.

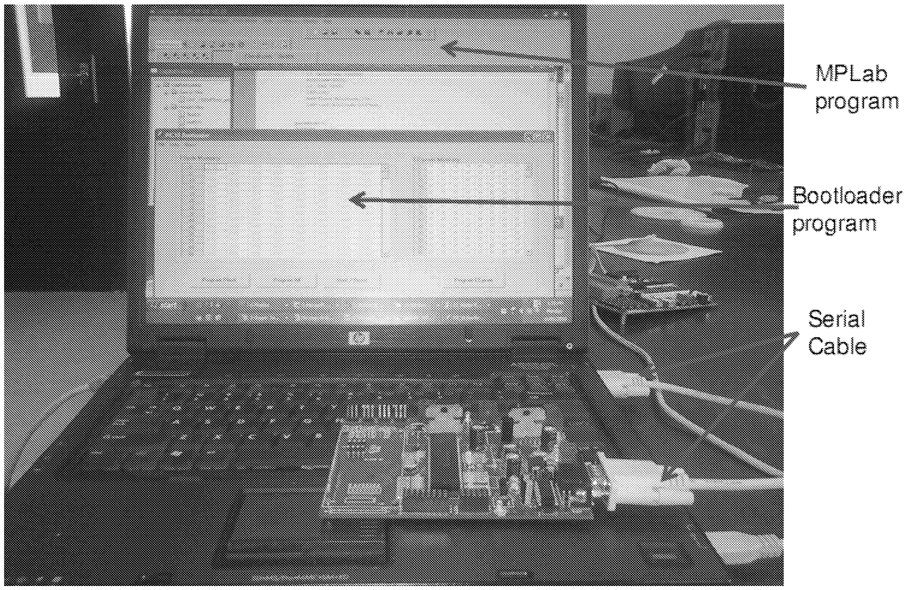

Figure 9.10: Microcontroller programming using MPLab development environment. This microcontroller is pre-loaded with a bootloader, therefore, it can be directly connected to the MPLab through the download cable

9.4 Example

Lets try to electronically implement the servo controller in Fig.7.13. The controller, which consists of a lag, a lead, and two gains is

$$\begin{aligned}\frac{U(s)}{E(s)} &= \frac{(s+0.95)}{(s+0.1)}1.85\frac{(s+10.8)}{(s+37)}129.3 \\ &= \frac{239.2(s^2+11.75s+10.26)}{s^2+37.1s+3.7} \\ &= \frac{64.6(s^2+11.75s+10.26)}{0.27s^2+10.03+1} \qquad (9.20)\end{aligned}$$

9.4.1 Analog Controller Synthesis

Therefore, control input $U(s)$ is given by

$$U(s) = 64.6[s^2+11.75s+10.26]E(s) - [0.27s^2+10.03s]U(s) \qquad (9.21)$$

which can be implemented as shown in Fig.9.11. This controller can be built with four unity-gain differentiators, five gains, three summing points, and one subtractor.

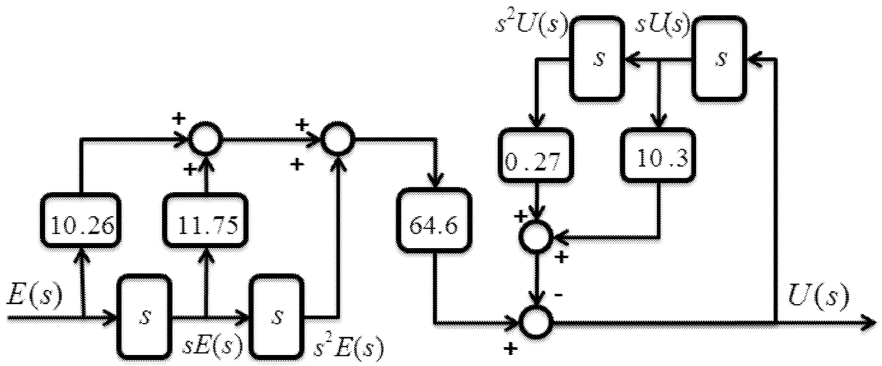

Figure 9.11: Construction of the servo controller in Fig.7.13

9.4.2 Digital Controller Synthesis

Controller in (9.20) can be written in time domain using (9.16) as

$$0.27\frac{d^2u(t)}{dt^2}+10.3\frac{du(t)}{dt}+u(t) = 64.6(\frac{d^2e(t)}{dt^2}+11.75\frac{de(t)}{dt}+10.26e(t)) \quad (9.22)$$

and discretize using (9.17), and (9.18) as follows

$$\begin{aligned}
0.27\frac{u_k - 2u_{k-1} + u_{k-2}}{T^2} &= 64.6\left(\frac{e_k - 2e_{k-1} + e_{k-2}}{T^2}\right. \\
+10.3\frac{u_k - u_{k-1}}{T} + u_k & \quad\left. +11.75\ \frac{e_k - e_{k-1}}{T} + 10.26e_k\right) \\
\left(\frac{0.27}{T^2} + \frac{10.3}{T} + 1\right) u_k &= 64.6\left\{\left(\frac{1}{T^2} + \frac{11.75}{T} + 10.26\right) e_k\right. \\
-\left(\frac{0.54}{T^2} + \frac{10.3}{T}\right) u_{k-1} + \frac{0.27}{T^2}u_{k-2} & \quad -\left(\frac{2}{T^2} + \frac{11.75}{T}\right) e_{k-1} + \frac{1}{T^2}e_{k-2}
\end{aligned} \quad (9.23)$$

Lets assume that we have our digital controller operating at 20Hz sampling rate (sampling period $T = 0.05$[s]), then (9.23) can be evaluated as follows

$$\begin{aligned}(108+206+1)u_k-(216+206)u_{k-1} &= 64.6\{(400+235+10.26)e_k \\ +108u_{k-2} &\quad -(800+235)e_{k-1}+400e_{k-2}\} \\ 315u_k-422u_{k-1}+108u_{k-2} &= 64.6\{645.26e_k-1035e_{k-1} \\ &\quad +400e_{k-2}\}\end{aligned} \tag{9.24}$$

which yields to the following discrete-time (digital) controller

$$u_k = 132.3e_k - 212.3e_{k-1} + 82e_{k-2} + 1.3u_{k-1} - 0.3u_{k-2} \tag{9.25}$$

This digital controller can be programmed onto a microcontroller as shown in Fig.9.12, microcontroller takes samples of the reference $r(t)$ and response $y(t)$ through its analog input terminals at every T[s] and calculates the control input $u(k)$, which is sent to the actuator through one of its output pins. Microcontrollers have built-in analog-to-digital converters (ADC) at the input pins, but they usually do not have digital to analog converters (DAC) at the output pins. If you want $u(k)$ to be an analog signal, then an external DAC should be used between the microcontroller output pin and the actuator.

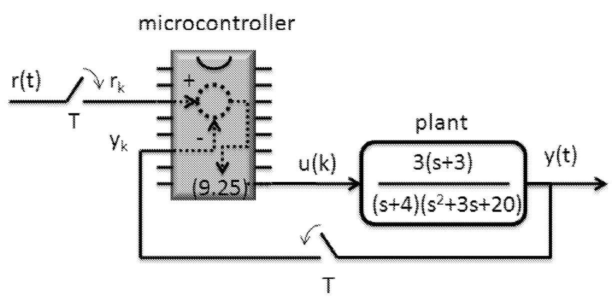

Figure 9.12: Use of a microcontroller to control the plant in Fig.7.13

9.5 Summary

The canonical form of the controller is useful in the controller implementation, which can be carried out either in analog (continuous-time) form, or digital (discrete-time) form. In analog implementation, the canonical form of the controller is synthesized using OpAmps. In digital implementation, the canonical form is discretized into the equivalent difference equation, and the resulting digital controller is programmed onto a microcontroller.